PERFORMANCE CHARACTERISTICS OF THE SWITCHED RELUCTANCE MOTOR IN ELECTRIC VEHICLE DURING ACCELERATION AT VARIABLE TERMINAL VOLTAGE

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***Abstract*— In this paper, the equations describing the performance of the electric vehicle are derived. Performance characteristics for each part in the vehicle system are obtained when the vehicle is accelerated under a variable terminal voltage while the turn on, turn off angles are constants.**

***Keywords*-Switched reluctance motor, SRM, electric vehicle, acceleration mode.**

I. INTRODUCTION

The internal combustion engine (ICE) vehicle at the present is a major source of urban pollution. According to figures released by the U.S. Environmental Protection Agency (EPA), conventional ICE vehicles currently contribute 40%–50% of ozone (nonmethane organic gases NMOG), 80%–90% of carbon monoxide (CO), and 50%–60% of air toxins (nitrogen oxides NOx) found in urban areas. Beside air pollution, the other main objection regarding ICE automobiles is their extremely low efficiency use of fossil fuel. Hence, the problem associated with ICE automobiles is threefold: environmental, economical, as well as political. These concerns have forced governments all over the world to consider alternative vehicle concepts [1]–[3].

Electric vehicles (EV) offer the most promising solutions to reduce vehicular emissions. EV constitute the only commonly known group of automobiles that qualified as zero emission vehicles (ZEVs). These vehicles use electric motors for propulsion and batteries as electrical energy storage devices [1,2].

Figure 1 shows The Drivetrain of the electric vehicle.

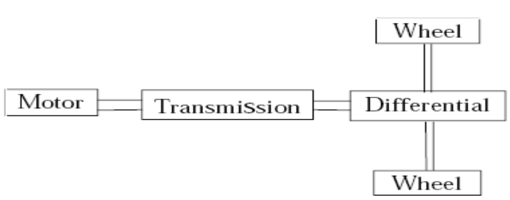

Figure 1. The EV Drivetrain

Switched reluctance motor (SRM) are perhaps the simplest of electrical machines. They consist of a stator with excitation windings and a magnetic rotor with saliency. Rotor conductors are not required because torque is produced by the tendency of the rotor to align with the stator produced flux wave in such a fashion as to maximize the stator flux linkages that result from a given applied stator current.

Due to simple and rugged motor construction, low weight, potentially low production cost, undemanding cooling, excellent torque–speed characteristics, high torque density, high operating efficiency, and inherent fault tolerance, switched reluctance motor (SRM) drives are emerging as an attractive solution for electric vehicle (EV) applications [4]–[6]. Traction performances of EVs depend on the performances of SRM drives. Hence, the excellent motoring operation of SRMs is important for EVs with high performances.

II. PERFORMANCE EQUATIONS OF THE EV WITH ACCELERATION MODE

To investigate the EV performance at acceleration it will be assumed that the vehicle is accelerated with the motor drive is fed from a variable DC voltage source, with a constant turn on and turn off angle.

The voltage equation of each phase winding can be expressed as:

$$v = i\,R + \frac{d\lambda\,(i,\theta)}{dt} \tag{1}$$

Neglecting the saturation of the magnetic circuit, the phase flux linkage can be expressed by:

$$\lambda\,(i,\theta) = L(\theta)\; i \tag{2}$$

By substitution from (2) into (1), the motor phase voltage can be written as:

$$v = i\,R + L(\theta)\,\frac{di}{dt} + i\,\frac{dL(\theta)}{dt} \tag{3}$$

Also (3) can be rewritten as:

$$v = i\,R + L(\theta)\,\frac{di}{d\theta}\,\frac{d\theta}{dt} + i\,\frac{dL(\theta)}{d\theta}\,\frac{d\theta}{dt} \quad (4)$$

At steady state, the motor speed can be determined as a function of the rotor position by:

$$\omega = \frac{d\theta}{dt} \quad (5)$$

where ω, θ are the motor speed in (elec. rad/s) and the rotor position in (elec. rad) respectively.

Substituting from (5) into (4), the motor phase voltage can be expressed as:

$$v = i\,R + L(\theta)\,\omega\,\frac{di}{d\theta} + i\,\omega\,\frac{dL(\theta)}{d\theta} \quad (6)$$

where the three terms of the above equation represents the resistive drop, the self and rotational EMF respectively.

Finally after rearranging (6), the motor phase voltage can be rewritten as:

$$v = \left(R + \omega\,\frac{dL(\theta)}{d\theta}\right) i + \omega\,L(\theta)\,\frac{di}{d\theta} \quad (7)$$

From Fig.2 the motor phase inductance can be represented as a function of the rotor position as:

$$\begin{aligned} L(\theta) &= L_u & 0 \le \theta \le \theta_i \\ L(\theta) &= k_1\,\theta - k_2 & \theta_i \le \theta \le \pi \\ L(\theta) &= -k_1\,\theta - k_3 & \pi \le \theta \le (2\pi - \theta_i) \\ L(\theta) &= L_u & (2\pi - \theta_i) \le \theta \le 2\pi \end{aligned} \quad (8)$$

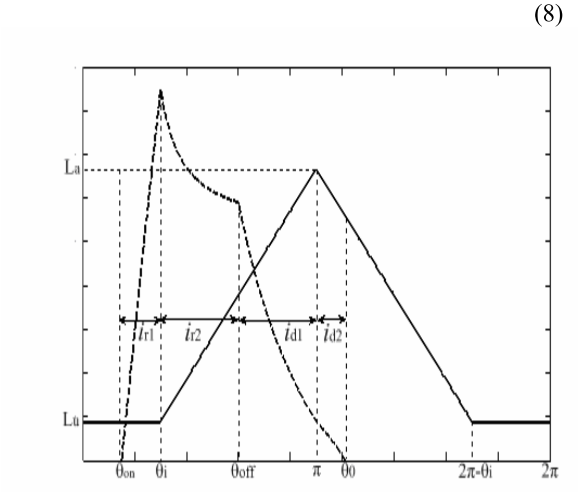

Figure 2. Motor phase inductance and current against the rotor position

The constants k_1, k_2, k_3 and θ_i are equal:

$$k_1 = \frac{L_a - L_u}{\pi - \theta_i}$$

$$k_2 = \frac{L_a\,\theta_i - L_u\,\pi}{\pi - \theta_i}$$

$$k_3 = \frac{-\pi\,(L_a - L_u) - L_a\,(\pi - \theta_i)}{\pi - \theta_i}$$

$$\theta_i = \pi - N_r\,\beta_r$$

According to the motor phase voltage there are two modes of operation of the SRM. At the first mode of operation the motor phase voltage is connected to the DC supply voltage Vs thus the motor phase current will be increases. At the second mode the applied voltage on the motor phase is the negative value of the DC supply voltage, thus the motor phase current will be decays to zero value.

Substituting from (8) into (7) taking into account the specified modes of operation, the motor phase current can be obtained for each rotor position range, included in Fig.2, as:

$$i_{r1}(\theta) = \frac{V_s}{R}\left[1 - e^{-(\theta - \theta_{on})\,R/(\omega\,L_u)}\right] \qquad \theta_{on} \le \theta \le \theta_i$$

$$i_{r2}(\theta) = \frac{V_s}{R + k_1\,\omega} + \left[i_{r1}(\theta_i) - \frac{V_s}{R + k_1\,\omega}\right]\left[\frac{k_1\,\theta_i - k_2}{k_1\,\theta - k_2}\right]^{\left(\frac{R + k_1\,\omega}{k_1\,\omega}\right)} \qquad \theta_i \le \theta \le \theta_{off}$$

$$i_{d1}(\theta) = \frac{-V_s}{R + k_1\,\omega} + \left[i_{r2}(\theta_{off}) + \frac{V_s}{R + k_1\,\omega}\right]\left[\frac{k_1\,\theta_{off} - k_2}{k_1\,\theta - k_2}\right]^{\left(\frac{R + k_1\,\omega}{k_1\,\omega}\right)} \qquad \theta_{off} \le \theta \le \pi$$

$$i_{d2}(\theta) = \frac{-V_s}{R - k_1\,\omega} + \left[i_{d1}(\pi) + \frac{V_s}{R - k_1\,\omega}\right]\left[\frac{-k_1\,\pi - k_3}{-k_1\,\theta - k_3}\right]^{\left(\frac{R - k_1\,\omega}{-k_1\,\omega}\right)} \qquad \pi \le \theta \le \theta_0 \quad (9)$$

Where these equations are derived to represent the following range for the turn on and turn off angle: $\theta_{on} < \theta_i$ and $\theta_{off} < \pi$.

Also θ_0 is the angle at which the motor phase current equal zero after decaying. This angle can be determined from (9) by putting ($i_{d2} = 0$ & $\theta = \theta_0$):

$$\theta_0 = \frac{1}{L_a - L_u}\left[\frac{-L_a\,(\pi - \theta_i)\left[(R - k_1\,\omega)\,i_{d1}(\pi) + V_s\right]^{k_4}}{V_s^{\,k_4}} + \pi(L_a - L_u) + L_a(\pi - \theta_i)\right] \quad (10)$$

Where the constant k_4 is equal

$$k_4 = \frac{-k_1\,\omega}{R - k_1\,\omega}$$

Therefore, the motor torque is expressed by:

$$T_e = \frac{1}{2}\, i^2\, \frac{dL(\theta)}{d\theta} \qquad (11)$$

The motor speed can be expressed in terms of the vehicle speed as:

$$\omega_m = m\,\frac{V_{veh}}{r_{wh}} \qquad (12)$$

where m is the gear ratio of the mechanical coupling between the motor and the axle of the vehicle wheels.

Assuming lossless transmission, the developed torque at the shaft of the wheel axle can be determined by;

$$T_{d\_wh} = m\,T_e \qquad (13)$$

The corresponding tractive force will, thus, be:

$$F_{TR} = \frac{T_{d\_wh}}{r_{wh}} \qquad (14)$$

The tractive force developed at the shaft of the wheel axle during acceleration can be expressed by:

$$F_{TR} = k_m\,M_{veh}\,\frac{dV_{veh}}{dt} + F_{RL} \qquad (15)$$

Where the road load force is [1,6]:

$$F_{RL} = C_0\,M_{veh}\,g + M_{veh}\,g\,\sin(\beta) + 0.5\,\rho\,C_D\,A_f V_{veh}^2 \qquad (16)$$

Thus, the load torque at the shaft of the wheel axle can be expressed by:

$$T_{wh} = F_{RL}\,r_{wh} + T_b \qquad (17)$$

Also, the load torque at the shaft of the motor axle can be expressed by:

$$T_L = \frac{T_{wh}}{m} \qquad (18)$$

Therefore from (15), the acceleration of the vehicle can be expressed by [4]:

$$\frac{dV_{veh}}{dt} = \frac{1}{k_m\,M_{veh}}\,(F_{TR} - F_{RL}) \qquad (19)$$

Also using the motor torque and speed, the motor output power can be expressed by:

$$P_{mo} = T_e\,\omega_m \qquad (20)$$

The motor power losses can be determined by;

$$P_{Loss} = n_{ph}\,I_{ph}^{\,2}\,R \qquad (21)$$

where I_{ph} is the average value of the motor phase current.

The motor excessive energy can be determined from:

$$E_{exc} = n_{ph}\,(I_{ph} - I_r)^2\,R\,dt \qquad (22)$$

where I_r, dt are the rated value of the motor phase current and the time step.

III. PRINCIPLE OF NUMERICAL SOLUTION

Starting from zero vehicle speed at constant turn on, turn off angle and certain terminal voltage, from (12) the motor speed would be equal to zero. From (9) and 10, the motor phase current can be determined. Then using the phase current into (11) the motor developed torque, T_e, can be obtained. Also using (13) the developed torque at the shaft of the wheel axle, T_{d_wh}, can be obtained. The corresponding tractive force, F_{TR}, can, thus, be obtained from (14). From (16), the road load force can be determined at this vehicle speed.

Using these values of the tractive and road load force into (19), the next vehicle speed can be obtained by integrating this equation numerically over an appropriate time step. For the second and following time steps of numerical solution, the corresponding motor speed is obtained from (12). Then (11) is used to obtain its motor developed torque and the corresponding tractive force is obtained from (14). This process continues until the vehicle reaches steady-state speed.

IV. SIMULATION RESULTS

The approach presented in (3), was applied using 4th order Runge-Kutta numerical method of integration. Several performance characteristics of the vehicle during acceleration, at variable terminal voltage, and fixed turn on and turn off angle using the data of the switched reluctance motor and vehicle given in Appendix (A) are obtained:

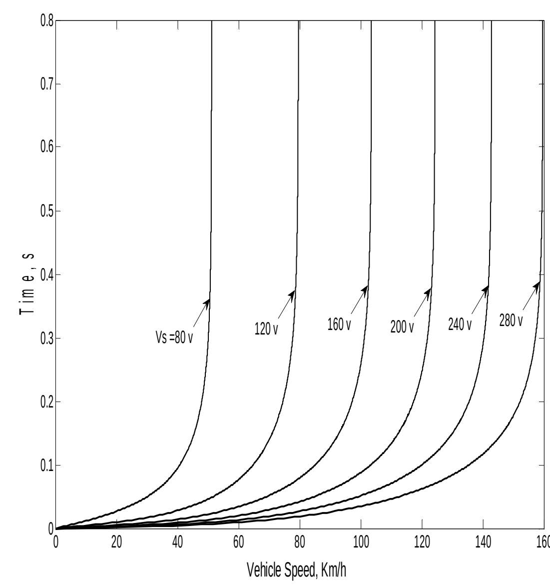

Figure 3. Vehicle speed versus time during acceleration.

By varying the terminal voltage at constant turn on and turn off angle (0° and 30° respectively) Fig.3 shows the variation of the vehicle speed throughout the acceleration period. From this figure it is clear that the vehicle reaches a higher final steady-state speed as the motor voltage increases.

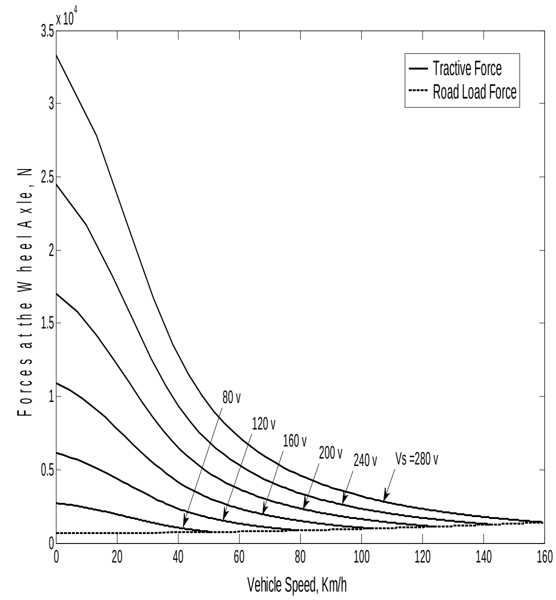

Figure 4. Tractive and resisting forces at wheel axle versus vehicle speed during acceleration.

From (14) and (16), the tractive and resisting forces, F_{TR} and F_{RL} respectively, are plotted against the vehicle speed during the acceleration period until steady-state conditions are reached, at the same values of the voltage used to obtain Fig.3, as shown in Fig.4.

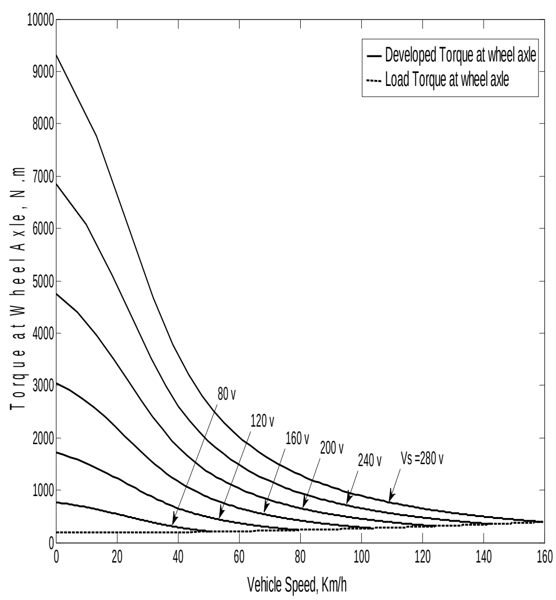

Figure 5. Torque at wheel axle versus vehicle speed during acceleration.

From Fig.4 it is clear that, for certain voltage values, the tractive force decreases and the resisting force increases as the vehicle speed increases up to steady-state speed at which the curves of the tractive and resisting forces are intersected. For a certain vehicle speed, the tractive force decreases while the resisting force is constant for the several values of voltage used when their values decrease.

From (13) and (17) the characteristics of the developed and load torque at the shaft of the wheel axle, T_{d_wh} and T_{wh}, are obtained against the vehicle speed, at different values of the motor terminal voltage, as shown in Fig.5. From this figure it is noticed that the developed and load torque applied on the wheel axle have the same shape as that of the corresponding tractive and resisting forces shown in Fig.4 and for the same vehicle speed the developed torque decreases as the used values of the voltage decrease.

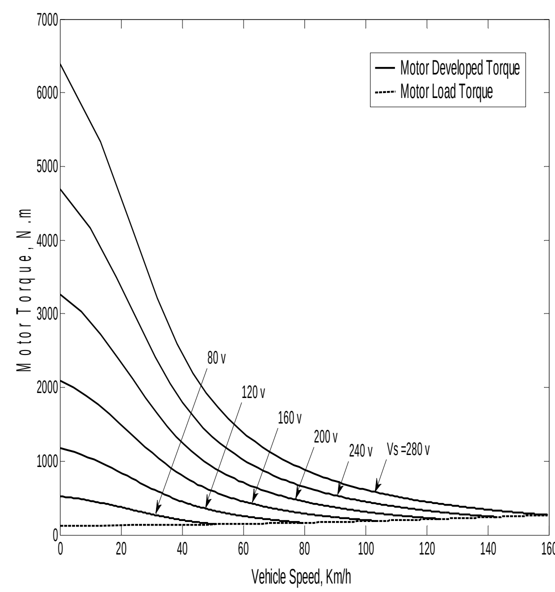

Figure 6. The motor torque versus vehicle speed during acceleration.

Using the values of the vehicle speed, which are obtained at different values of motor voltage, into (9) and (10) then substituting into (11), the characteristics of the motor developed torque, T_e , can be obtained. Also from (18), the motor load torque, T_L , can be determined. Then the motor developed torque and load torque are drawn versus the vehicle speed, during acceleration until steady-state conditions are reached, as shown in Fig.6.

From this figure it is clear that, for certain voltage values, the motor developed torque decreases and the load torque increases as the vehicle speed increases up to steady-state speed at which the curves of the developed and load torques are intersected. For a certain vehicle speed, the developed torque decreases while the load torque is constant for the several values of voltage used when their values decrease.

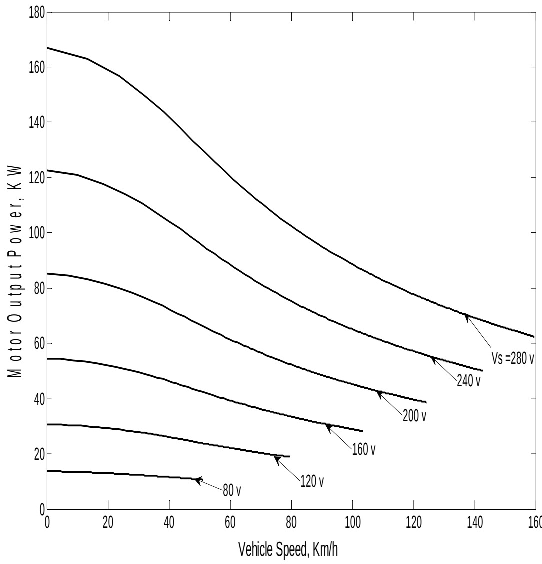

Figure 7. Motor output power versus vehicle speed during acceleration.

Using the motor developed torque, calculated from (11), and the motor speed, calculated from (12), into (20) the characteristics of the motor output power, P_{mo}, can be obtained against the vehicle speed, at different values of the motor voltage, as shown in Fig.7.

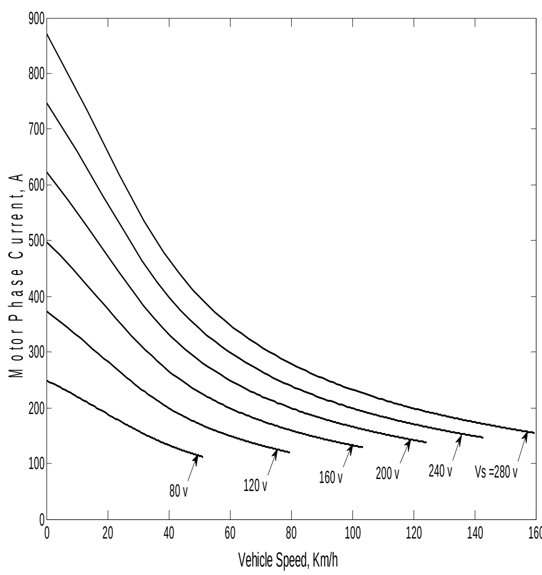

Figure 8. Motor phase current versus vehicle speed during acceleration.

From Fig.7 for certain values of motor voltage, the motor output power decreases as the vehicle speed increases. Also, at certain vehicle speed the developed power has higher values at higher voltages.

Using the values of the vehicle speed, which are obtained, into (9) and (10), the motor phase current can be determined and plotted against vehicle speed, for different values of the motor terminal voltage, as shown in Fig.8. From this figure it is clear that the motor phase current decrease as the vehicle speed increases and for the same vehicle speed the current decreases as the used values of the voltage decrease.

Using the values of the motor phase current, which are obtained at different values of motor voltage, into (21), the characteristics of the motor power losses, P_{Loss}, can be drawn versus the vehicle speed, during acceleration until steady-state conditions are reached, as shown in Fig.9. From this figure it is noticed that the motor power losses decrease as the vehicle speed increases and for the same vehicle speed the motor losses decreases as the used values of the voltage decrease.

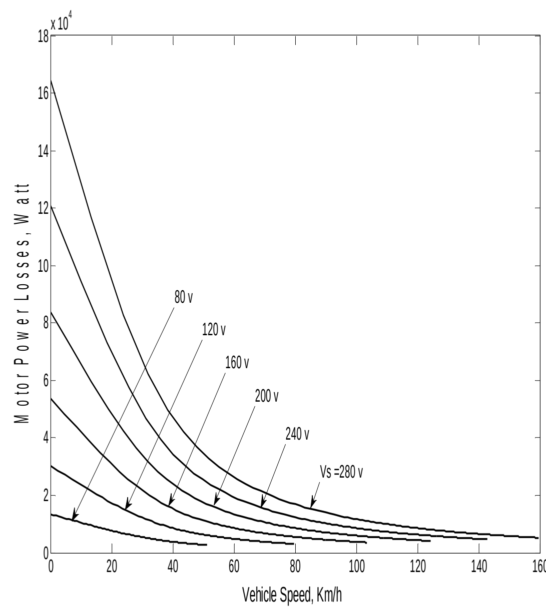

Figure 9. Motor power losses versus vehicle speed during acceleration.

Using the predetermined motor phase current at different values of the motor terminal voltage, into (22) the motor excessive energy, E_{exc}, can be computed and plotted as shown in Fig.10.

From this figure it is clear that at certain values of voltage, the motor excessive energy increases as the vehicle accelerates then decreases to zero before reaching the final steady state speed. For any vehicle speed, the motor excessive energy will have larger values for higher values of the terminal voltage.

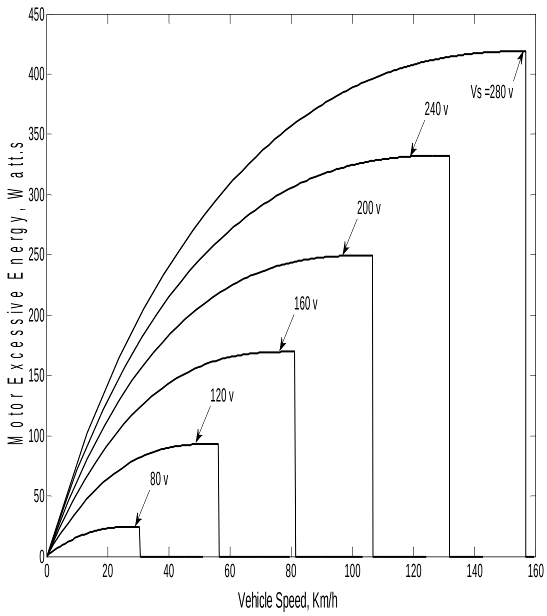

Figure 10. Motor excessive energy versus vehicle speed during acceleration.

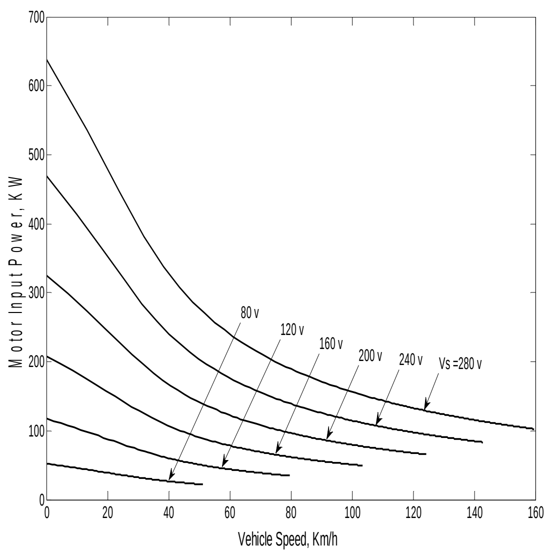

Figure 11. Motor input power versus vehicle speed during acceleration.

Multiplying the predetermined motor phase current, which is obtained at different values of the motor terminal voltage from (9) and (10), by the motor terminal voltage, the motor input power, P_{m_in}, can be also computed and plotted as shown in Fig.11.

From this figure it is clear that at certain values of voltage, the motor input power decreases as the vehicle accelerates and reaches a constant value at steady state. For any vehicle speed, the motor input power will have larger values for higher values of the terminal voltage.

At certain values of the motor terminal voltage, From (6) the motor rotational and self EMF can be calculated respectively at different values of the vehicle speed. Then the rotational and self EMF are plotted versus vehicle speed as shown in Fig.12 and 13 respectively.

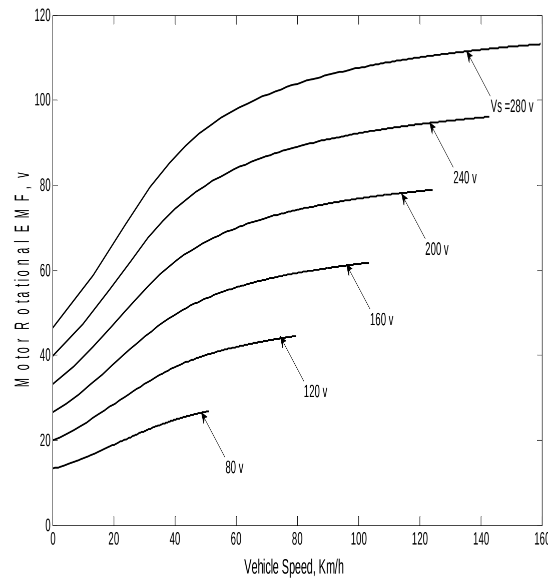

Figure 12. Motor rotational EMF versus vehicle speed during acceleration.

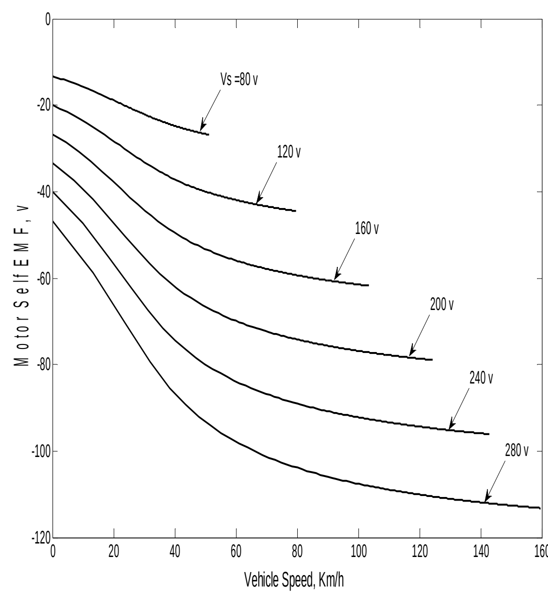

Figure 13. Motor self EMF versus vehicle speed during acceleration.

From Fig.12 it is clear that at a constant terminal voltage, the motor rotational EMF increase as the vehicle accelerates and then at a constant vehicle speed the rotational EMF decreases as the terminal voltage decreases.

From Fig.13 it is clear that at a constant terminal voltage, the motor self EMF decrease as the vehicle accelerates and then at a constant vehicle speed the self EMF will have larger values for lower values of the terminal voltage.

V. CONCLUSIONS

From the performance characteristics of the EV operating in the acceleration mode, the following is observed, as the motor terminal voltage increases:

- the vehicle reaches a higher final steady-state speed.
- the tractive force increases while the resisting force is constant for a certain vehicle speed.
- the motor developed torque increases for a certain vehicle speed while the load torque is constant.
- the motor developed power has higher values at certain vehicle speed.
- the motor phase current will have larger values for any vehicle speed.
- the motor power losses has higher values at certain vehicle speed.
- the motor excessive energy has larger values at certain vehicle speed.
- the motor input power increases at a constant vehicle speed.
- the motor rotational EMF has larger values at a constant vehicle speed.
- the motor self EMF has lower values at a constant vehicle speed.

Also for certain operating values of the motor voltage :

- the tractive force decreases and the resisting force increases as the vehicle speed increases up to steady-state speed at which the tractive and resisting forces are equal.
- the motor developed torque decreases and the load torque increases as the vehicle speed increases up to steady-state speed.
- the motor developed power decreases as the time increases until the vehicle reaches steady-state speed at which the developed power becomes constant.
- the motor phase current decreases as the vehicle accelerates and reaches a constant value at steady state.
- the motor power losses decreases as the vehicle accelerates.
- the motor excessive energy increases as the vehicle accelerates then decrease to zero value near the final steady-state speed.
- the motor input power decreases as the vehicle accelerates.
- the motor rotational EMF increases as the vehicle speed increases up to steady-state speed.
- the motor self EMF decreases as the vehicle accelerates.

LIST OF SYMBOLS

A_f	Equivalent frontal area of the vehicle in m^2.
C_0	Coefficient of rolling resistance.
C_D	Aerodynamic drag coefficient.
E_{exc}	Motor excessive energy.
F_{RL}	Road load force in N.
F_{TR}	Tractive force in N.
g	Gravitational acceleration constant in m/s^2.
i	Motor phase current
I_{ph}	The average value of the motor phase current in A.
I_r	Rated value of the motor phase current in A.
$k_{1\ldots}$ K_4	Constants.
k_m	Rotational inertia coefficient.
L	Motor phase inductance.
L_a	Aligned inductance in mH.
L_u	Unaligned inductance in mH.
m	The gear ratio of the mechanical coupling between the motor and the axle of the vehicle wheels.
M_{veh}	Total mass of the vehicle in kg.
n_{ph}	Number of motor phases.
N_r	Number of rotor poles.
P_{Loss}	Motor power losses in W.
P_{m-in}	Motor input power in W.
P_{mo}	Motor output power in W.
R	Motor phase resistance in ohm.
r_{wh}	Radius of the wheel in m.
T_b	Frictional brake torque in Nm.
T_{d_wh}	The developed torque at the shaft of the wheel axle
T_e	Motor developed torque in Nm.
T_L	Load torque at the shaft of the motor axle in Nm.
T_{wh}	Load torque at the shaft of the wheel axle in Nm.
v	Motor phase voltage in v.
V_s	DC supply voltage in v.
V_{veh}	Vehicle speed in km/h.
β	Road grade angle.
β_r	Rotor poles arc.
θ	Rotor position in elec.rad.

θ_0	The angle at which the motor phase current equal zero after decaying.
θ_{off}	Turn off angle.
θ_{on}	Turn on angle.
λ	Motor phase flux linkage.
ρ	Air density in kg/m^3.
ω	Motor speed in elec.rad/s.
ω_m	Motor speed in rad/s.

APPENDIX (A)

Data of the SRM

P_r =60 kW, V_r=280 V, R=0.072 ohm, L_a=3.334 mH, L_u=0.445 mH, Nr=4, Ns=6, n_{ph} =3, Bs=Br=π/6, j=0.3, b=0.0183,n_r=2214 rpm

Vehicle dynamic parameters

ρ =1.225 kg/m^3, C_D =0.3, A_f =2 m^2, M_{veh} =1500 kg, r_{wh} =0.2794 m, Tb=0, $V_{veh\text{-}max}$ =160 km/h, V_f =100 km/h, km =1.08, C_0=0.01, g =9.81m/s^2, m= 1.4575, β=2°. η_{tmw} =1.00.

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